

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

ATKINSON SCIENCE

$$e^{i\pi} = -1$$
$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \frac{y u_\tau}{\nu}$$
$$E_b = \sigma T^4$$

THEORY GUIDE

Humidity Calculator Web Application

Keith Atkinson

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Atkinson Science welcomes your comments on this Theory Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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1 Introduction

You can find the Atkinson Science Humidity Calculator web application at the web address <https://atkinsonscience.co.uk/WebApps/Construction/HumidityCalculator.aspx>. There is a user guide that you can download at the same address.

The humidity calculator web application calculates the properties of moist air when the temperature, pressure and moisture content of the air are typical of those found in air conditioning systems. The underlying theory can be found in text books on engineering thermodynamics, such as Refs. [1] and [2]. Moist air is a mixture of dry air and water vapour, and the properties of the mixture are determined by regarding it as a mixture of two perfect gases. The water vapour constituent in the mixture differs from the dry air in the fact that it can condense out of the mixture at ordinary environmental temperatures.

2 Properties of water

2.1 p - v - T

When water is in the vapour phase in atmospheric air we can regard the vapour as a perfect gas. The primary properties of the vapour, pressure p [Pa], specific volume v [m³ kg⁻¹], and temperature T [K], are related to each other by a simple analytical equation, known as the *perfect gas law*. For water and substances in general it is not possible to write a simple analytical equation for p , v and T that applies to all possible phases (solid, liquid and gas). We must present the relationship between p , v and T as a chart or in tabular form. Ref. [3] presents tables of thermodynamic properties of water in the liquid, solid and gas phases.

Figure 1 is a chart showing the temperature of the different phases of water (ice, liquid and vapour) against specific volume for different values of pressure. The curve coloured blue is known as the *saturated liquid line*, the curve coloured red is known as the *saturated vapour line*, and the point at which the two curves meet is known as the *critical point*. The horizontal line coloured green along which the pressure is 0.006112 bar is known as the *triple line*.

Suppose a volume of ice is enclosed in a cylinder by a piston, as shown in Figure 2. The weight of the piston maintains the pressure constant at 1 atm. The ice is initially at state (1) on the isobar labelled $p = 1 \text{ atm.} = 1.01325 \text{ bar}$ in Figure 1.

If the cylinder is heated slowly so that the temperature of the ice remains uniform then we can track the expansion of the ice along the isobar until it reaches state (2) at which the temperature is 273.15 K (0°C). If there is no friction in the process, so that it is reversible, then the change in internal energy u [kJ kg⁻¹] of the ice is related to the heat supplied $Q_{1 \rightarrow 2}$ [kJ kg⁻¹] and the work done in overcoming the downward force of the piston $W_{1 \rightarrow 2}$ [kJ kg⁻¹] by

$$Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} = (u_2 - u_1)$$

Note that W is taken as positive if work is done *by* the system. Since the process is reversible we can write

$$W_{1 \rightarrow 2} = \int_1^2 p dv$$

and since p is constant,

$$W_{1 \rightarrow 2} = p(v_2 - v_1) = p_2 v_2 - p_1 v_1$$

so

$$Q_{1 \rightarrow 2} = (u_2 + p_2 v_2) - (u_1 + p_1 v_1)$$

The specific enthalpy h [kJ kg⁻¹] is defined as $u + pv$, so the heat supplied is

$$Q_{1 \rightarrow 2} = (h_2 - h_1)$$

Figure 1 Isobars on $T-v$ diagram for water. Not to scale

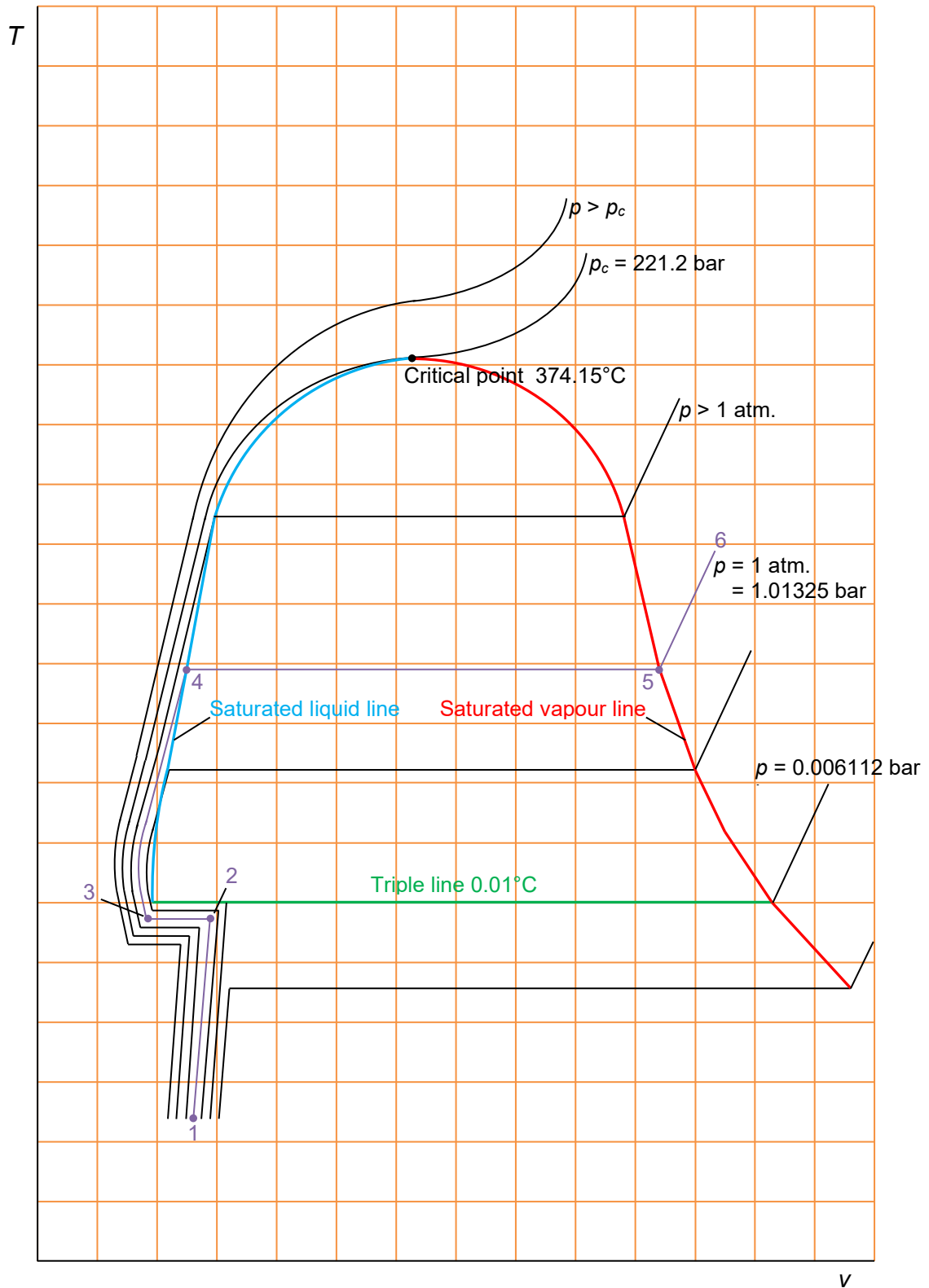
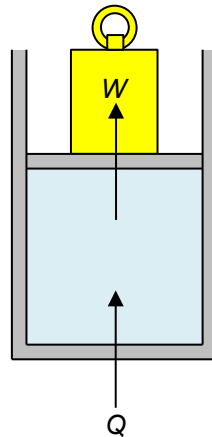


Figure 2 Constant pressure process

The temperature at point (2) is 273.15 K (0°C). This is the melting temperature of ice at 1 atm. Further heating does not raise the temperature of the ice but causes it to melt. Melting is complete at point (3). The heat needed to convert all of the ice to water is known as the *latent heat of fusion* or *latent enthalpy of fusion*. Further heating causes the temperature of the liquid to rise until it reaches point (4) at which the temperature is 373.15 K (100°C). This is the boiling temperature of water at 1 atm. Further heating causes a second phase change at constant temperature in which the water is converted to vapour. Vaporisation is complete at point (5). The heat needed to convert all of the liquid to vapour is known as the *latent heat of vaporisation* or *latent enthalpy of vaporisation*.

Properties on the saturated liquid line and the saturated vapour line are given the subscripts *f* and *g*, respectively, so

$$Q_{4 \rightarrow 5} = (h_5 - h_4) = (h_g - h_f) = h_{fg}$$

At 1 atm., $h_f = 419.1 \text{ kJ kg}^{-1}$ and $h_g = 2675.8 \text{ kJ kg}^{-1}$ (Ref. [3]), so the latent heat of vaporisation h_{fg} is $2256.7 \text{ kJ kg}^{-1}$.

To the left of the saturated liquid line the water exists as a liquid, while to the right of the saturated vapour line the water exists as a vapour. Between the two lines the water is a two-phase mixture of liquid and vapour or *wet vapour*. The vapour to the right of the saturated vapour line is known as *unsaturated vapour* or *superheated steam*. Further heating causes the temperature of the vapour to rise. Between points (5) and (6), we can assume that p , v and T are related by the equation of state for a perfect gas:

$$pv = RT \quad (2.1)$$

where $R \text{ [J kg}^{-1} \text{ K}^{-1}]$ is the specific gas constant for water vapour. The specific gas constant of a particular gas can be determined from the universal gas constant $R_u \text{ [J kmol}^{-1} \text{ K}^{-1}]$ and the molar mass $M \text{ [kg kmol}^{-1}]$ of the gas:

$$R = \frac{R_u}{M}$$

The molar masses of elements and compounds can be obtained from the NIST Chemistry WebBook, Ref. [4]. According to NIST, the most accurate value of the universal gas constant R_u is currently 8314.4598 J kmol⁻¹ K⁻¹. NIST gives the molar mass of water vapour as 18.0153 kg kmol⁻¹, so the specific gas constant of water vapour is

$$R = \frac{R_u}{M} = \frac{8314.4598}{18.0153} = 461.522 \text{ J kg}^{-1}\text{K}^{-1}$$

When the pressure is reduced to 0.006112 bar the melting and boiling temperatures become equal and the change of phase, ice-water-vapour, is represented by the triple line, coloured green in Figure 1. Only along this line can ice, water and vapour coexist in thermodynamic equilibrium in a closed vessel. The temperature at which this occurs is taken to be a fixed point on the absolute temperature scale and is *defined* to be 273.16 K (0.01°C). The thermodynamic properties internal energy u and entropy s are *defined* to be zero for saturated liquid at the *triple point*; that is, at the intersection between the saturated liquid line and the triple line. Thus,

$$p = 0.006112 \text{ bar}, T = 0.01^\circ\text{C}, u_f = 0$$

$$p = 0.006112 \text{ bar}, T = 0.01^\circ\text{C}, s_f = 0$$

The specific volume of saturated liquid v_f at the triple point is 0.0010002 m³ kg⁻¹ (Ref. [3]). The specific enthalpy of saturated liquid at the triple point is therefore

$$\begin{aligned} h_f &= u + pv = 0 + 0.006112 \text{ bar} \times 100,000 \text{ Pa bar}^{-1} \times 0.0010002 \text{ m}^3 \text{ kg}^{-1} \times 0.001 \text{ kJ J}^{-1} \\ &= 0.0006112 \text{ kJ kg}^{-1} \end{aligned}$$

The specific enthalpy of saturated vapour h_g at the triple point is 2500.8 kJ kg⁻¹ (Ref. [3]), so the latent heat of vaporisation h_{fg} along the triple line is

$$h_{fg} = h_g - h_f = 2500.8 - 0.0006112 = 2500.8 \text{ kJ kg}^{-1}$$

2.2 Specific enthalpies h_f , h_{fg} and h_g

The specific enthalpy of water in the liquid phase is only weakly dependent on pressure, so to the left of the saturated liquid line we can write

$$h(p, T) = h(T) = h_f(T)$$

There is no simple analytical equation for $h_f(T)$ or for the latent heat of vaporisation $h_{fg}(T)$. The authors of Ref. [5] propose the following simple empirical formulas for $h_f(\theta)$ and $h_{fg}(\theta)$, where θ is the temperature in Celsius:

$$h_f(\theta) = a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5 \quad (2.2)$$

$$h_{fg}(\theta) = b_1 + b_2\theta + b_3\theta^{1.5} + b_4\theta^{2.5} + b_5\theta^3 \quad (2.3)$$

The specific enthalpy of saturated vapour $h_g(\theta)$ is calculated simply by adding $h_f(\theta)$ and $h_{fg}(\theta)$.

The values of the constants a_1 to a_6 and b_1 to b_5 in formulas (2.2) and (2.3) are given in Table 1. The formulas are valid for θ from 0.01 to 150°C. Over this range the formula for h_f deviates from the tabulated values in Ref. [6] by no more than ± 0.013 kJ kg⁻¹. At 15°C the tabulated value of h_f is 62.9 kJ kg⁻¹, so at this temperature the deviation corresponds to a level of uncertainty of $\pm 0.02\%$. When compared with the tabulated values in Ref. [6], the estimated uncertainty in the formula for h_{fg} over the range 0.01 to 150°C is $\pm 0.09\%$.

Table 1 Values of constants in empirical formulas for h_f and h_{fg}

h_f		h_{fg}	
Constant	Value	Constant	Value
a_1	-2.844699×10^{-2}	b_1	2500.304
a_2	4.211925	b_2	-2.2521025
a_3	-1.017034×10^{-3}	b_3	-0.021465847
a_4	1.311054×10^{-5}	b_4	3.1750136×10^{-4}
a_5	-6.756469×10^{-8}	b_5	$-2.8607959 \times 10^{-5}$
a_6	1.724481×10^{-10}		

2.3 Specific heats C_v and C_p for water vapour

The *specific heat at constant volume* C_v of a substance is defined by the relation

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (2.4)$$

where the right-hand side of (2.4) is the change in internal energy with temperature at constant volume.

The *specific heat at constant pressure* C_p of a substance is defined by the relation

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (2.5)$$

where the right-hand side of (2.5) is the change in enthalpy with temperature at constant pressure.

We can show mathematically that for a perfect gas the internal energy u is a function of temperature only, i.e.

$$u = u(T) \quad (2.6)$$

Combining the definition of enthalpy $h = u + pv$ with the equation of state of a perfect gas $pv = RT$ gives

$$h = u + RT \quad (2.7)$$

Since R is a constant and $u = u(T)$, it follows that the enthalpy of a perfect gas is also a function of temperature only:

$$h = h(T) \quad (2.8)$$

Since u and h depend on T alone, C_v and C_p must also depend on T alone, and the partial derivatives in the equations for C_v and C_p can be replaced by ordinary derivatives. Thus, for a perfect gas:

$$C_v(T) = \frac{du}{dT} \quad \text{or} \quad du = C_v(T)dT \quad (2.9)$$

and

$$C_p(T) = \frac{dh}{dT} \quad \text{or} \quad dh = C_p(T)dT \quad (2.10)$$

Since $h = u + RT$,

$$dh = du + RdT$$

Substituting (2.9) and (2.10) gives

$$C_p(T)dT = C_v(T)dT + RdT$$

and so

$$C_p(T) = C_v(T) + R \quad (2.11)$$

For a *calorically perfect gas*, C_v and C_p are both constant, and we can integrate (2.9) and (2.10) immediately to obtain

$$(u_2 - u_1) = C_v(\theta_2 - \theta_1) \quad (2.12)$$

and

$$(h_2 - h_1) = C_p(\theta_2 - \theta_1) \quad (2.13)$$

where θ is the temperature in Celsius. Eqn. (2.11) becomes

$$C_p = C_v + R \quad (2.14)$$

To determine the specific enthalpy of water vapour h at a given temperature, we can either:

- assume $h(T) = h_g(T)$ and determine $h(T)$ from an empirical formula for $h_g(T)$, or
- assume C_p is constant and use Eqn. (2.13) with $h_1 = h_g = 2500.8 \text{ kJ kg}^{-1}$ at $\theta_1 = 0.01^\circ\text{C}$.

In the humidity calculator web application, we have chosen option (b). This approach is consistent with the way in which the specific enthalpy of the dry air is calculated in a mixture of dry air and water vapour. In practice, along the saturated vapour line, C_p increases from 1.86 to 1.88 $\text{kJ kg}^{-1} \text{K}^{-1}$ as θ increases from 0.01 to 25°C (Ref. [3]). In the web application we have taken C_p to be 1.86 $\text{kJ kg}^{-1} \text{K}^{-1}$, the value at 0.01°C. Neglecting the term $\theta_1 = 0.01^\circ\text{C}$ in (2.13), the specific enthalpy of water vapour is therefore

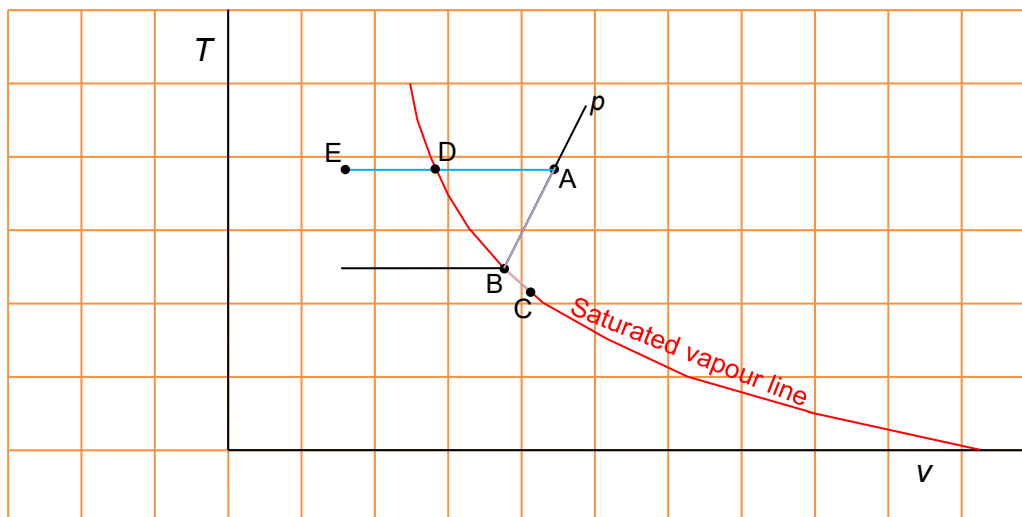
$$h[\text{kJ kg}^{-1}] = 2500.8 + 1.86\theta \quad (2.15)$$

3 Condensation

Suppose a volume of water vapour is at state A on the isobar labelled p in Figure 3. If the temperature of the vapour is lowered, the location of the state of the vapour will move along the isobar until it reaches state B on the saturated vapour line. Any further decrease in temperature will cause condensation to occur. The temperature at which condensation occurs is called the saturation vapour temperature at the pressure p . The condensation will cause a lowering of the vapour pressure and so the location of the state of the vapour will move down the saturated vapour line to C.

Now suppose the volume of the vapour at state A is reduced so that its pressure increases at constant temperature. The location of the state of the vapour will move along the horizontal line coloured blue representing the temperature T until it reaches state D on the saturated vapour line. Any further decrease in volume will cause condensation to occur. The pressure at which condensation occurs is called the saturation vapour pressure at the temperature T . Any further decrease in volume will cause the location of the state to move further along the constant temperature line to E.

Figure 3 Condensation of vapour at constant pressure and at constant temperature



Ref. [7] provides the following relationships for the saturation vapour pressure p_g [Pa] and the temperature θ [C] for temperatures commonly found in buildings:

For $\theta \geq 0$:

$$p_g = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) \quad (3.1)$$

For $\theta < 0$:

$$p_g = 610.5 \exp\left(\frac{21.875 \times \theta}{265.5 + \theta}\right) \quad (3.2)$$

The relationship for $\theta < 0$ gives the saturation vapour pressure over ice.

These relationships are particularly useful because they can be inverted to give the saturation vapour temperature θ_g [C] in terms of the vapour pressure p_v :

For $p_v \geq 610.5\text{Pa}$:

$$\theta_g = \frac{237.3 \ln\left(\frac{p_v}{610.5}\right)}{17.269 - \ln\left(\frac{p_v}{610.5}\right)} \quad (3.3)$$

For $p_v < 610.5\text{Pa}$:

$$\theta_g = \frac{265.5 \ln\left(\frac{p_v}{610.5}\right)}{21.875 - \ln\left(\frac{p_v}{610.5}\right)} \quad (3.4)$$

4 Properties of mixtures of air and water vapour

4.1 Equation of state

A perfect gas has the equation of state

$$pV = mRT \quad (4.1)$$

where p [Pa] is the pressure of a mass m [kg] of gas in a volume V [m³], T [K] is the temperature of the gas, and R [J kg⁻¹ K⁻¹] is a constant that is unique to the gas and is called the *specific gas constant* or simply the *gas constant*.

Dalton's law of partial pressure states that the pressure exerted by each gas in a mixture of gases is independent of the presence of the others, and that the total pressure of the mixture is equal to the sum of the partial pressures. Furthermore, we can show that if Dalton's law holds then each constituent in the mixture obeys the equation of state (4.1).

Air is a mixture, so it is not covered by the NIST chemistry database. The Engineering Toolbox, Ref. [8], gives the molar mass of dry air as 28.9647 kg kmol⁻¹. The specific gas constant of dry air is therefore

$$R = \frac{R_u}{M} = \frac{8314.4598}{28.9647} = 287.055 \text{ J kg}^{-1}\text{K}^{-1}$$

As stated in Section 2.1, the specific gas constant of water vapour is 461.522 J kg⁻¹ K⁻¹.

4.2 Moisture content

The *specific humidity* or *moisture content* ω is the ratio of the mass of water vapour m_v to the mass of dry air m_a in a mixture of air and water vapour:

$$\omega = \frac{m_v}{m_a} \quad (4.2)$$

We can use Dalton's law to express ω in terms of the partial pressures of the mixture constituents. Suppose a mixture of water vapour and air occupies a volume V [m³] at temperature T [K]. Regarding the water vapour as a perfect gas, the partial pressure of the vapour in the mixture p_v [Pa], is given by

$$p_v V = m_v R_v T \quad (4.3)$$

where m_v [kg] is the mass of vapour in the mixture and R_v [J kg⁻¹ K⁻¹] is the specific gas constant of the vapour.

Similarly, the partial pressure of the dry air in the mixture p_a [Pa], is given by

$$p_a V = m_a R_a T \quad (4.4)$$

where m_a [kg] is the mass of vapour in the mixture and R_a [J kg⁻¹ K⁻¹] is the specific gas constant of dry air.

Substituting (4.3) and (4.4) into (4.2) gives

$$\omega = \frac{m_v}{m_a} = \frac{p_v V R_a T}{p_a V R_v T} = \frac{R_a p_v}{R_v p_a}$$

Now $R_a = 287.055 \text{ J kg}^{-1} \text{ K}^{-1}$ and $R_v = 461.522 \text{ J kg}^{-1} \text{ K}^{-1}$, and so

$$\omega = 0.622 \frac{p_v}{p_a} \quad (4.5)$$

Since $p_a = p - p_v$, where p is the atmospheric (barometric) pressure, we can write (4.5) as

$$\omega = 0.622 \frac{p_v}{p - p_v} \quad (4.6)$$

4.3 Relative humidity

The *relative humidity* ϕ is the ratio of the mass of water vapour m_v in a volume V of mixture at temperature T to the highest mass of water vapour that the volume can contain at the same temperature before condensation occurs. The state at which the highest mass of water vapour occurs will be located on the saturated vapour line at the temperature T , so

$$\phi = \frac{m_v}{m_g} \quad (4.7)$$

The water vapour can be regarded as a perfect gas, so

$$\phi = \frac{p_v V R_v T}{p_g V R_v T} = \frac{p_v}{p_g} \quad (4.8)$$

where p_g is the saturation vapour pressure of the water vapour at the mixture temperature T .

4.4 Dew point

The *dew point* T_{dp} is the temperature at which condensation begins if the mixture is cooled at constant pressure. In other words, T_{dp} is the saturation temperature corresponding to the vapour pressure.

4.5 Specific volume of dry air

The specific volume of dry air v_a in a mixture of air and water vapour is

$$v_a = \frac{V}{m_a} \quad (4.9)$$

where m_a [kg] is the mass of air in volume V [m³] of mixture. Applying the perfect gas law,

$$v_a = \frac{R_a T}{p_a} \quad (4.10)$$

If the temperature and relative humidity of the mixture are known then the partial pressure of the vapour p_v can be calculated from (4.8) and (4.3) or (4.4). Then the partial pressure of the air p_a can be calculated from the atmospheric pressure p , $p_a = p - p_v$, and v_a can be obtained from (4.10).

The specific volume of dry air is useful in calculations involving air conditioning systems. If we know the volume flow rate of moist air \dot{V} [$\text{m}^3 \text{s}^{-1}$] through the system then we can calculate the mass flow rate of dry air \dot{m}_a from

$$\dot{m}_a = \frac{\dot{V}}{v_a} \quad (4.11)$$

4.6 Enthalpy

The enthalpy h [kJ kg^{-1}] used in air conditioning engineering is the specific enthalpy of moist air per kilogramme of dry air:

$$h = \frac{H_a + H_v}{m_a} = \frac{m_a h_a + m_v h_v}{m_a} = h_a + \omega h_v \quad (4.12)$$

We assumed that the dry air was a calorically perfect gas, so the specific enthalpy the of dry air h_a [kJ kg^{-1}] is determined from

$$h_{a,2} - h_{a,1} = C_{p,a}(\theta_2 - \theta_1) \quad (4.13)$$

where $C_{p,a}$ is a constant. For engineering calculations in which θ_1 and θ_2 have typical environmental values we can take $C_{p,a}$ to be $1.005 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and take $h_{a,1} = 0 \text{ kJ kg}^{-1}$ at $\theta_1 = 0^\circ\text{C}$. Eqn. (4.13) then becomes

$$h_a [\text{kJ kg}^{-1}] = 1.005 \theta \quad (4.14)$$

The specific enthalpy of water vapour h_v [kJ kg^{-1}] is determined from Eqn. (2.15).

4.7 Wet-bulb temperature

Air conditioning engineers need a method of determining the relative humidity ϕ and specific humidity (moisture content) ω of moist air from quantities that can be measured easily, such as temperature and pressure. The amount of vapour in the air can be determined easily from the *wet bulb* temperature.

The wet-bulb temperature is the temperature measured by a thermometer whose bulb has been wrapped in a wick moistened with water. Moisture will evaporate from the wick and after a period of time a state of equilibrium will be reached in which the sensible heat gain to the water is equal to the latent heat loss from it and the water has attained a steady temperature θ' , termed the wet-bulb temperature.

Ref. [9] gives the following equation which expresses this condition:

$$(h_c + h_r)A(\theta - \theta') = \alpha Ah_{fg}(p'_g - p_v)$$

where h_c and h_r are coefficients for convection and radiation heat transfer to the wick, respectively, A is the surface area of the wick, h_{fg} is the latent heat of vaporisation of water at temperature θ' , θ is the dry-bulb temperature, α is a diffusion coefficient for the molecules of vapour leaving the wick, and p'_g is the saturation vapour pressure at θ' . The equation can be simplified to give an expression for the vapour pressure of the moist air in terms of the measured values of dry-bulb and wet-bulb temperature:

$$p_v = p'_g - \left[\frac{h_c + h_r}{\alpha Ah_{fg}} \right] (\theta - \theta')$$

The term in square brackets is a function of atmospheric pressure and temperature. This equation is usually expressed in the following form, which is termed the psychrometric equation:

$$p_v = p'_g - pC(\theta - \theta') \quad (4.16)$$

where p is the atmospheric (barometric) pressure and C is a constant. C has the values shown in Table 2, reproduced from Ref. [9]. In the web application the psychrometer is assumed to be of the sling or aspirated type.

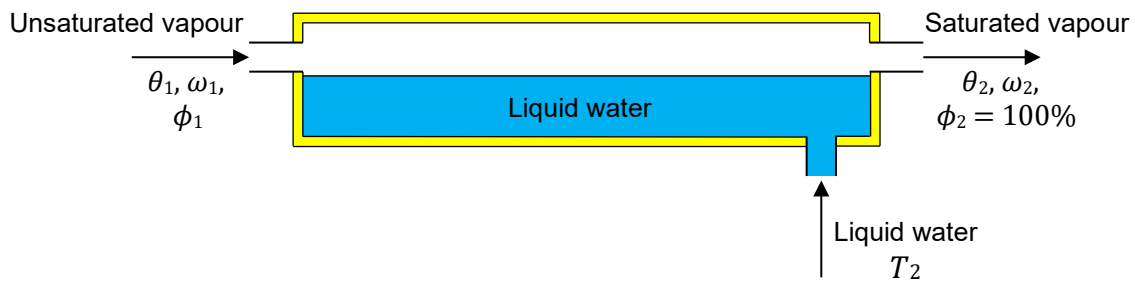
Table 2 Values of the constant C in the psychrometric equation

Psychrometer	wet-bulb $\geq 0^\circ\text{C}$	wet-bulb $< 0^\circ\text{C}$
Screen	$7.99 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$	$7.20 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$
Sling or aspirated	$6.66 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$	$5.94 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$

4.8 Adiabatic saturation temperature

Another method of determining the amount of moisture in the air is to subject the air to an *adiabatic saturation process*. The system needed to carry out the process consists of a long insulated channel containing a pool of water, as shown in Figure 4. A steady flow of moist air with known mass flow rate \dot{m}_1 and known temperature T_1 enters at one end of the channel. As the air flows over the water, the water evaporates and mixes with the air. The temperature of the moist air falls as it provides part of the latent heat of vaporisation to the water. If the channel is long enough then the air leaving the channel will be saturated ($\phi = 100\%$). The temperature θ_2 of the saturated air is known as the *adiabatic saturation temperature*. Make-up water at temperature θ_2 is added to the channel at the rate of evaporation \dot{m}_f .

Figure 4 Adiabatic saturation process



The adiabatic saturation process can be regarded as a steady-flow process with no heat or work transfer to the system. The mass flow rate of dry air \dot{m}_a passing through the system is constant. The mass flow rate of vapour leaving the system is

$$\dot{m}_{v2} = \dot{m}_{v1} + \dot{m}_f$$

or

$$\dot{m}_{a2}\omega_2 = \dot{m}_{a1}\omega_1 + \dot{m}_f$$

and since \dot{m}_a is constant, the mass flow rate of make-up water \dot{m}_f is

$$\dot{m}_f = \dot{m}_a(\omega_2 - \omega_1) \quad (4.17)$$

There is no heat or work transfer, so the rate at which energy leaves the system is

$$\dot{H}_2 = \dot{H}_1 + \dot{H}_{f2}$$

or

$$\dot{m}_a h_2 = \dot{m}_a h_1 + \dot{m}_f h_{f2} \quad (4.18)$$

where h_1 and h_2 are the enthalpies of the moist air per unit mass of dry air at the inlet and the outlet of the channel, respectively, and h_{f2} is the enthalpy per unit mass of water at temperature θ_2 .

Substituting \dot{m}_f from (4.17) into (4.18) gives

$$\dot{m}_a h_2 = \dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f2}$$

and after dividing \dot{m}_a ,

$$h_2 = h_1 + (\omega_2 - \omega_1) h_{f2} \quad (4.19)$$

Substituting (4.12) for h_1 and h_2 into this equation gives

$$(h_{a2} + \omega_2 h_{v2}) = (h_{a1} + \omega_1 h_{v1}) + (\omega_2 - \omega_1) h_{f2} \quad (4.20)$$

We can write the specific enthalpy of the dry air h_a using Eqn. (4.14):

$$h_a [\text{kJ kg}^{-1}] = C_{p,a} \theta_a = C_{p,a} \theta \quad (4.14)$$

where $C_{p,a} = 1.005 \text{ kJ kg}^{-1} \text{ K}^{-1}$ is the specific heat at constant pressure of dry air, θ_a [°C] is the temperature of the dry air, which is also the temperature of the mixture θ [°C]. Substituting (4.14) into (4.20) gives

$$(C_{p,a} \theta_2 + \omega_2 h_{v2}) = (C_{p,a} \theta_1 + \omega_1 h_{v1}) + (\omega_2 - \omega_1) h_{f2}$$

so

$$\omega_1 = \frac{C_{p,a}(\theta_2 - \theta_1) + \omega_2(h_{v2} - h_{f2})}{(h_{v1} - h_{f2})} \quad (4.21)$$

The vapour specific enthalpies h_{v1} and h_{v2} can be obtained from (2.15). The specific enthalpy of the water h_{f2} can be obtained from (2.2). The relative humidity of the moist air leaving the channel ϕ_2 is 100%, so from (4.6),

$$\omega_2 = 0.622 \frac{p_{g2}}{p - p_{g2}}$$

where p_{g2} is the saturation vapour pressure at θ_2 .

5 Examples

5.1 Condensation

In cold weather, condensation occurs on the inside surfaces of windows because the temperature of the windows is below the room temperature. If a room contains air at 20°C and 75% relative humidity, what must be the window temperature for moisture to begin condensing on the inside of the windows?

From Eqn. (3.1), the saturation vapour pressure p_g of water vapour at 20°C is

$$p_g = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 20}{237.3 + 20}\right) = 2337.0 \text{ Pa}$$

Note that the tables of saturated water and steam p. 2 in Ref. [3] also give p_g as 2337 Pa.

From Eqn. (4.8), the partial pressure of the vapour p_v is

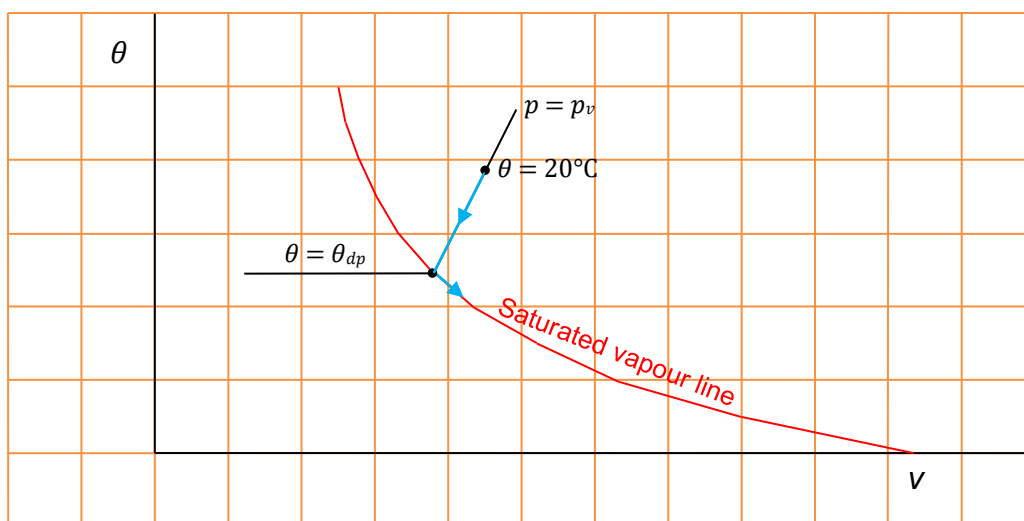
$$p_v = \phi p_g = 0.75 \times 2337.0 = 1752.7 \text{ Pa}$$

Moisture will begin to condense on the windows when the temperature of the air reaches the dew point θ_{dp} . The process is shown on the θ - v chart for water in Figure 5. The dew point is the saturation vapour temperature corresponding to the vapour pressure 1752.7 Pa. From Eqn. (3.3), θ_{dp} is

$$\theta_{dp} = \frac{237.3 \ln\left(\frac{p_v}{610.5}\right)}{17.269 - \ln\left(\frac{p_v}{610.5}\right)} = \frac{237.3 \ln\left(\frac{1752.7}{610.5}\right)}{17.269 - \ln\left(\frac{1752.7}{610.5}\right)} = 15.4^\circ\text{C}$$

Note that interpolating linearly between the saturation vapour pressures 1704 Pa and 1817 Pa in the tables of saturated water and steam p. 2 in Ref. [3] also gives θ_{dp} as 15.4°C.

Figure 5 θ - v chart showing condensation process



5.2 Wet-bulb temperature

Calculate the relative humidity of moist air at a barometric pressure is 101.325 Pa if the measured dry-bulb temperature is 20°C and the wet-bulb temperature measured by a sling psychrometer is 15°C.

The vapour pressure p_v [Pa] in the moist air is given by Eqn. (4.16) (the psychrometric equation):

$$p_v = p'_g - pC(\theta - \theta')$$

where θ [C] is the dry-bulb temperature, θ' [C] is the wet-bulb temperature, p [Pa] is the atmospheric (barometric) pressure, p'_g [Pa] is the saturation vapour pressure at the wet-bulb temperature, and C [°C⁻¹] is a constant. For a sling psychrometer and a wet-bulb temperature greater than or equal to 0°C, Table 2 gives C as $6.66 \times 10^{-4} \text{ °C}^{-1}$.

From Eqn. (3.1), the saturation vapour pressure p'_g of water vapour at the wet-bulb temperature 15°C is

$$p'_g = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 15}{237.3 + 15}\right) = 1704.4 \text{ Pa}$$

The vapour pressure p_v in the moist air is therefore

$$p_v = 1704.4 - 101325 \times 6.66 \times 10^{-4} \times (20 - 15) = 1367.0 \text{ Pa}$$

From Eqn. (3.1), the saturation vapour pressure p_g at the dry-bulb temperature 20°C of the moist air is

$$p_g = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 20}{237.3 + 20}\right) = 2337.0 \text{ Pa}$$

From Eqn. (4.8), the relative humidity ϕ [%] of the moist air is therefore

$$\phi = \frac{p_v}{p_g} = \frac{1367.0}{2337.0} \times 100\% = 58.5\%$$

5.3 Adiabatic saturation temperature

When a stream of moist air undergoes an adiabatic saturation process, its temperature falls from 20°C to 13.5°C. Calculate the initial relative humidity of the moist air.

Eqn. (4.21) relates the initial moisture content ω_1 of moist air which undergoes an adiabatic saturation process to the final adiabatic saturation temperature θ_2 [°C]:

$$\omega_1 = \frac{C_{p,a}(\theta_2 - \theta_1) + \omega_2(h_{v2} - h_{f2})}{(h_{v1} - h_{f2})} \quad (4.21)$$

θ_1 [°C] is the initial temperature of the moist air, ω_2 is the final moisture content of the moist air, $C_{p,a} = 1.005 \text{ kJ kg}^{-1} \text{ K}^{-1}$ is the specific heat at constant pressure of dry air, h_{v1} and h_{v2} [kJ kg⁻¹] are the specific enthalpies of water vapour at θ_1 and θ_2 , respectively, and h_{f2} [kJ kg⁻¹] is the specific enthalpy of the water at θ_2 .

Specific enthalpy of water vapour

From Eqn. (2.15), the specific enthalpy of water vapour at 20°C, h_{v1} [kJ kg⁻¹], is

$$h_{v1} = 2500.8 + 1.86\theta = 2500.8 + 1.86 \times 20 = 2538 \text{ kJ kg}^{-1}$$

and the specific enthalpy of water vapour at 13.5°C, h_{v2} [kJ kg⁻¹], is

$$h_{v2} = 2500.8 + 1.86\theta = 2500.8 + 1.86 \times 13.5 = 2526 \text{ kJ kg}^{-1}$$

Specific enthalpy of liquid water

The specific enthalpy of water h_{f2} [kJ kg⁻¹] is given by Eqn. (2.2):

$$h_f(\theta) = a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5$$

For $\theta_2 = 13.5^\circ\text{C}$, h_{f2} is

$$\begin{aligned} h_{f2} &= -2.844699 \times 10^{-2} + 4.211925 \times 13.5 - 1.017034 \times 10^{-3} \times 13.5^2 \\ &+ 1.311054 \times 10^{-5} \times 13.5^3 - 6.756469 \times 10^{-8} \times 13.5^4 + 1.724481 \times 10^{-10} \times 13.5^5 \\ &= 56.7 \text{ kJ kg}^{-1} \end{aligned}$$

Note that interpolating linearly between the temperatures 13°C and 14°C in the tables of saturated water and steam p. 2 in Ref. [3] gives h_{f2} at 13.5°C as 56.7 kJ kg⁻¹.

Moisture content at the adiabatic saturation temperature

From Eqn. (3.1), the saturation vapour pressure p_{g2} of water vapour at 13.5°C is

$$p_{g2} = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 13.5}{237.3 + 13.5}\right) = 1547 \text{ Pa}$$

Note that interpolating linearly between the temperatures 13°C and 14°C in the tables of saturated water and steam p. 2 in Ref. [3] gives p_{g2} at 13.5°C as 1547 Pa.

The relative humidity of the moist air ϕ_2 at the end of the adiabatic saturation process is 100%, so from Eqn. (4.6),

$$\omega_2 = 0.622 \frac{p_{g2}}{p - p_{g2}} = 0.622 \times \frac{1547}{101325 - 1547} = 0.009644 \text{ kg vapour kg}^{-1} \text{ dry air}$$

Initial moisture content of the moist air

Substituting the values of θ_1 , θ_2 , ω_2 , $C_{p,a}$, h_{v1} , h_{v2} and h_{f2} into Eqn. (4.21) gives

$$\begin{aligned} \omega_1 &= \frac{1.005 \times (13.5 - 20) + 0.009644 \times (2526 - 56.7)}{(2538 - 56.7)} \\ &= 0.006965 \text{ kg vapour kg}^{-1} \text{ dry air} \end{aligned}$$

Initial relative humidity of the moist air

The moist air is initially at 20°C. From Eqn. (3.1), the initial saturation vapour pressure p_{g1} of the water vapour is

$$p_{g1} = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 20}{237.3 + 20}\right) = 2337 \text{ Pa}$$

Note that the tables of saturated water and steam p. 2 in Ref. [3] give p_{g1} as 2337 Pa.

From Eqn. (4.6), the initial vapour pressure p_{v1} in the moist air is

$$p_{v1} = \frac{p \omega_1}{0.622 + \omega_1} = \frac{101325 \times 0.006965}{0.622 + 0.006965} = 1122 \text{ Pa}$$

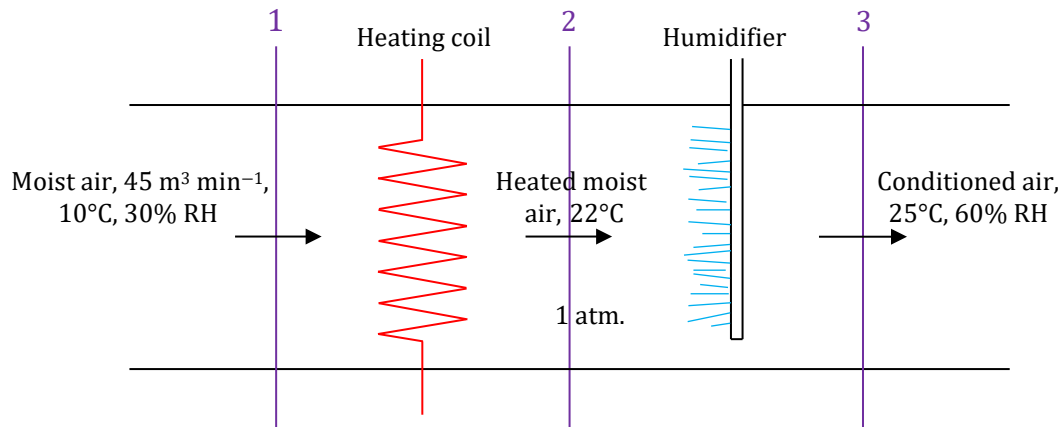
From Eqn. (4.8), the initial relativity of the moist air ϕ_1 is

$$\phi_1 = \frac{p_v}{p_g} \times 100\% = \frac{1122}{2337} \times 100\% = 48.0\%$$

5.4 Heating and humidifying

An air conditioning system is to take in outdoor air at 10°C and 30% relative humidity at a steady rate of 45 m³ min⁻¹ and to condition it to 25°C and 60% relative humidity. The outdoor air is first heated to 22°C in the heating section and then humidified by the injection of hot steam in the humidifying section. Assuming the entire process takes place at a pressure of 1 atm., determine (a) the rate of heat supply in the heating section and (b) the mass flow rate of steam required in the humidifying section.

Figure 6 Heating and humidifying system



Mass flow rate of dry air

The saturation vapour pressure at station 1, p_{g1} , can be found from Eqn. (3.1):

$$p_{g1} = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 10}{237.3 + 10}\right) = 1227 \text{ Pa}$$

and from Eqn. (4.8), the vapour pressure p_{v1} is

$$p_{v1} = \phi_1 p_{g1} = 0.30 \times 1227 = 368 \text{ Pa}$$

The partial pressure of the dry air p_a is therefore

$$p_a = p - p_v = 101325 - 368 = 100957 \text{ Pa.}$$

and from Eqn. (4.10), the specific volume of the dry air v_a is

$$v_a = \frac{R_a T}{p_a} = \frac{287.055 \times 283.15}{100957} = 0.8051 \text{ m}^3 \text{ kg}^{-1}$$

From Eqn. (4.11), the mass flow rate of dry air through the air conditioning system \dot{m}_a is therefore

$$\dot{m}_a = \frac{\dot{V}}{v_a} = \frac{45}{0.8051} = 55.9 \text{ kg min}^{-1}$$

Heat supply rate

From Eqn. (4.6), the moisture content ω_1 at station 1 is

$$\omega_1 = 0.622 \frac{p_{v1}}{p - p_{v1}} = 0.622 \times \frac{368}{101325 - 368} = 0.002267$$

and from Eqn. (4.2), the mass flow rate of water vapour \dot{m}_{v1} at station 1 is

$$\dot{m}_{v1} = \omega_1 \dot{m}_a = 0.002267 \times 55.9 = 0.1267 \text{ kg min}^{-1}$$

The enthalpy h [kJ kg⁻¹] used in air conditioning engineering is the specific enthalpy of moist air per kilogramme of dry air, which is determined from Eqn. (4.12):

$$h = h_a + \omega h_v$$

where h_a and h_v are the specific enthalpies of the air and the water vapour in the moist air, respectively. Both quantities are function of temperature only. We can determine h_{a1} from Eqn. (4.13) and h_{v1} from Eqn. (2.15):

$$h_{a1} = 1.005\theta = 1.005 \times 10 = 10.05 \text{ kJ kg}^{-1}$$

$$h_{v1} = 2500.8 + 1.86\theta = 2500.8 + 1.86 \times 10 = 2519 \text{ kJ kg}^{-1}$$

and so

$$h_1 = h_{a1} + \omega_1 h_{v1} = 10.05 + 0.002267 \times 2519 = 15.76 \text{ kJ kg}^{-1} \text{ dry air}$$

The moist air is heated to 22°C in the heating section, so h_{a2} and h_{v2} are

$$h_{a2} = 1.005\theta = 1.005 \times 22 = 22.11 \text{ kJ kg}^{-1}$$

$$h_{v2} = 2500.8 + 1.86\theta = 2500.8 + 1.86 \times 22 = 2542 \text{ kJ kg}^{-1}$$

The moisture content does not change through the heating section, so $\omega_2 = \omega_1$ and

$$h_2 = h_{a2} + \omega_2 h_{v2} = 22.11 + 0.002267 \times 2542 = 27.87 \text{ kJ kg}^{-1} \text{ dry air}$$

The energy equation for the heating process can be written as

$$\dot{Q}_{1 \rightarrow 2} = \dot{m}_a (h_2 - h_1)$$

so the heat supplied in the heating section is

$$\dot{Q}_{1 \rightarrow 2} = 55.9 \times (27.87 - 15.76) = 677 \text{ kJ min}^{-1}$$

Mass flow rate of steam

The saturation vapour pressure at station 3, p_{g3} , can be found from Eqn. (3.1):

$$p_{g3} = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 25}{237.3 + 25}\right) = 3166 \text{ Pa}$$

and from Eqn. (4.8), the vapour pressure p_{v3} is

$$p_{v3} = \phi_3 p_{g3} = 0.60 \times 3166 = 1900 \text{ Pa}$$

From Eqn. (4.6), the moisture content ω_3 at station 3 is

$$\omega_3 = 0.622 \frac{p_{v3}}{p - p_{v3}} = 0.622 \times \frac{1900}{101325 - 1900} = 0.01189$$

and from Eqn. (4.2), the mass flow rate of vapour \dot{m}_{v3} leaving the humidifying section is

$$\dot{m}_{v3} = \omega_3 \dot{m}_a = 0.01189 \times 55.9 = 0.6647 \text{ kg min}^{-1}$$

For conservation of mass in the air conditioning system,

$$\dot{m}_{v2} = \dot{m}_{v1}$$

and

$$\dot{m}_{v3} = \dot{m}_{v2} + \dot{m}_{w3}$$

The mass flow rate of steam required \dot{m}_{w3} is therefore

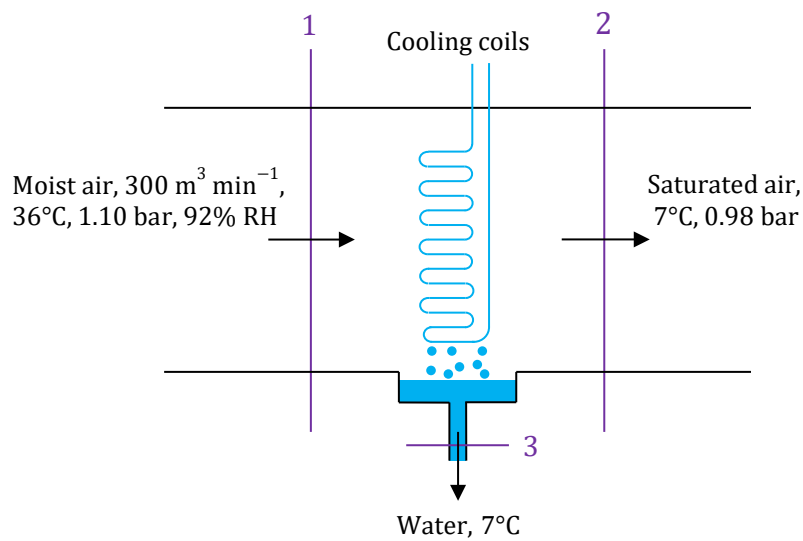
$$\dot{m}_{w3} = \dot{m}_{v3} - \dot{m}_{v1} = 0.6647 - 0.1267 = 0.5380 \text{ kg min}^{-1}$$

5.5 Cooling and dehumidifying

An air conditioning system takes in $300 \text{ m}^3 \text{ min}^{-1}$ of moist air at 36°C , 1.10 bar and 92% relative humidity. (a) Find the specific humidity of the mixture and the mass of water vapour and dry air entering the air conditioning system per minute.

The moist air is passed over refrigerator coils in order to dehumidify it and leaves the refrigerator chamber with a pressure and temperature of 0.98 bar and 7°C . The condensate also leaves the chamber at 7°C . (b) Determine the final specific humidity of the mixture, the mass of water condensing on the cooling coils per minute, and the rate of heat transfer to the cooling coils.

Figure 7 Cooling and dehumidifying system



(a) From Eqn. (3.1), the saturation vapour pressure at station 1, p_{g1} , is

$$p_{g1} = 610.5 \exp\left(\frac{17.269 \times \theta}{237.3 + \theta}\right) = 610.5 \exp\left(\frac{17.269 \times 36}{237.3 + 36}\right) = 5937 \text{ Pa}$$

and from Eqn. (4.8), the vapour pressure p_{v1} is

$$p_{v1} = \phi_1 p_{g1} = 0.92 \times 5937 = 5462 \text{ Pa}$$

From Eqn. (4.6), the specific humidity (moisture content) at station 1 is

$$\omega_1 = 0.622 \frac{p_{v1}}{p - p_{v1}} = 0.622 \times \frac{5462}{110000 - 5462} = 0.003250$$

From Eqn. (4.3) the mass flow rate of vapour in $300 \text{ m}^3 \text{ min}^{-1}$ of moist air is

$$\dot{m}_{v1} = \frac{p_{v1} \dot{V}_1}{R_v T_1} = \frac{5462 \times 300}{461.522 \times 309.15} = 11.48 \text{ kg min}^{-1}$$

From Eqn. (4.4) the mass flow rate of dry air in $300 \text{ m}^3 \text{ min}^{-1}$ of moist air is

$$\dot{m}_{a1} = \frac{p_{a1}\dot{V}_1}{R_a T_1} = \frac{(p - p_{v1})\dot{V}_1}{R_a T_1} = \frac{(110000 - 5462) \times 300}{287.055 \times 309.15} = 353.4 \text{ kg min}^{-1}$$

(b) We can assume that the moist air leaves the refrigerator chamber with a relative humidity of 100%.

The vapour pressure at station 2 is therefore

$$p_{v2} = p_{g2} = 610.5 \exp\left(\frac{17.269 \times \theta_2}{237.3 + \theta_2}\right) = 610.5 \exp\left(\frac{17.269 \times 7}{237.3 + 7}\right) = 1001 \text{ Pa}$$

and the specific humidity (moisture content) at station 2 is

$$\omega_2 = 0.622 \frac{p_{v2}}{p - p_{v2}} = 0.622 \times \frac{1001}{98000 - 1001} = 0.006419$$

For conservation of mass in the air conditioning system,

$$\dot{m}_{a2} = \dot{m}_{a1}$$

and

$$\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_{w3}$$

Using the first of these two equations, the mass flow rate of vapour leaving the refrigerator chamber is

$$\dot{m}_{v2} = \omega_2 \dot{m}_{a2} = \omega_2 \dot{m}_{a1} = 0.006419 \times 353.4 = 2.27 \text{ kg min}^{-1}$$

Using the second of the two equations, the mass flow rate of water is

$$\dot{m}_{w3} = \dot{m}_{v1} - \dot{m}_{v2} = 11.48 - 2.27 = 9.21 \text{ kg min}^{-1}$$

The heat transfer across the cooling coils is

$$\dot{Q}_{1 \rightarrow 2} = (\dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2} + \dot{m}_{w3} h_{w3}) - (\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1})$$

Applying the equations of conservation of mass gives

$$\begin{aligned} \dot{Q}_{1 \rightarrow 2} &= \dot{m}_{a1}(h_{a2} - h_{a1}) + \dot{m}_{v2}(h_{v2} - h_{v1}) + \dot{m}_{w3}(h_{w3} - h_{v1}) \\ &= \dot{m}_{a1} C_{p,a}(\theta_2 - \theta_1) + \dot{m}_{v2} C_{p,v}(\theta_2 - \theta_1) + \dot{m}_{w3}(h_{w3} - h_{v1}) \end{aligned}$$

The temperature of the condensate is 7°C , so from Eqn. (2.2) the specific enthalpy of the condensate h_{w3} is

$$\begin{aligned} h_{w3} &= h_{f3} = a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5 \\ &= -2.844699 \times 10^{-2} + 4.211925 \times 7 - 1.017034 \times 10^{-3} \times 7^2 \\ &\quad + 1.311054 \times 10^{-5} \times 7^3 - 6.756469 \times 10^{-8} \times 7^4 + 1.724481 \times 10^{-10} \times 7^5 \\ &= 29.41 \text{ kJ kg}^{-1} \end{aligned}$$

From Eqn. (2.15), the specific enthalpy of the water vapour at station 1 h_{v1} is

$$h_{v1} = 2500.8 + 1.86\theta_1 = 2500.8 + 1.86 \times 36 = 2568 \text{ kJ kg}^{-1}$$

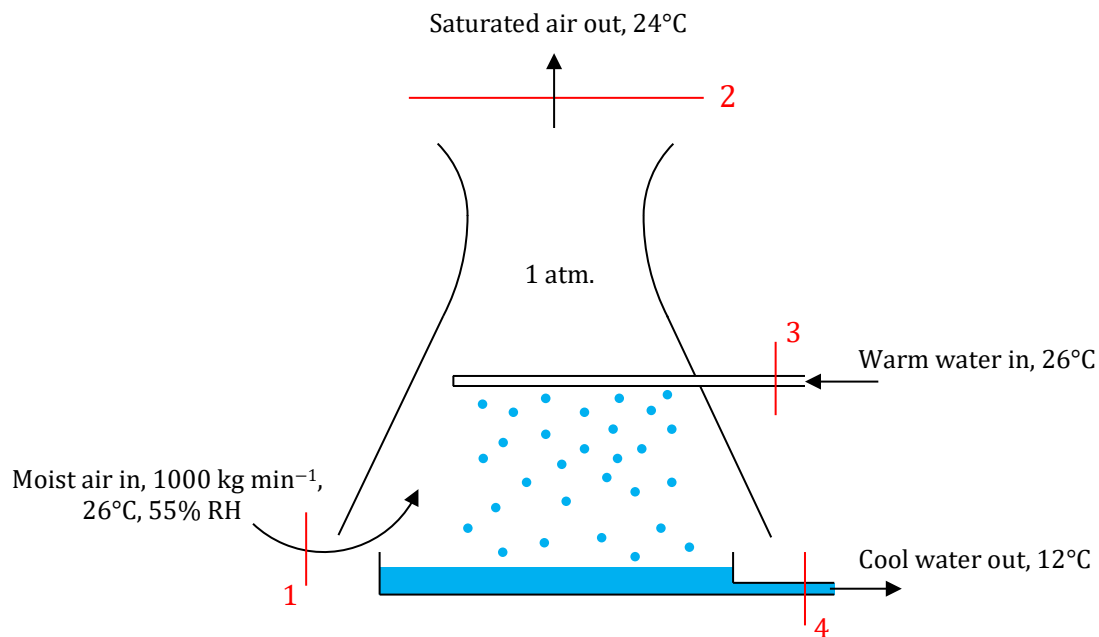
The heat transfer across the cooling coils is therefore

$$\begin{aligned}\dot{Q}_{1 \rightarrow 2} &= 353.4 \times 1.005 \times (7 - 36) + 2.27 \times 1.86 \times (7 - 36) + 9.21 \times (29.41 - 2568) \\ &= -33800 \text{ kJ min}^{-1}\end{aligned}$$

5.6 Cooling tower

1000 kg min⁻¹ of turbine condenser cooling water is sprayed into a natural draught cooling tower at a temperature of 26°C and the water returns to the condenser at 12°C. Air is drawn into the tower at 15°C with a relative humidity of 55% and leaves the tower saturated at 24°C. Calculate the mass flow rate of moist air into the tower and the rate of loss of cooling water by evaporation. It may be assumed that the total pressure throughout the tower is 1 atm.

Figure 8 Cooling tower inlets and outlets



Moisture content at station 1

At the air inlet station 1, the saturation vapour pressure p_{g1} is

$$p_{g1} = 610.5 \exp\left(\frac{17.269 \times \theta_1}{237.3 + \theta_1}\right) = 610.5 \exp\left(\frac{17.269 \times 15}{237.3 + 15}\right) = 1704 \text{ Pa}$$

the vapour pressure p_{v1} is

$$p_{v1} = \phi_1 p_{g1} = 0.55 \times 1704 = 937 \text{ Pa}$$

and the moisture content ω_1 is

$$\omega_1 = 0.622 \frac{p_{v1}}{p - p_{v1}} = 0.622 \times \frac{937}{101325 - 937} = 0.005806$$

Moisture content at station 2

At the air outlet station 2 the relative humidity is 100% so the vapour pressure p_{v2} is

$$p_{v2} = p_{g2} = 610.5 \exp\left(\frac{17.269 \times \theta_2}{237.3 + \theta_2}\right) = 610.5 \exp\left(\frac{17.269 \times 24}{237.3 + 24}\right) = 2982 \text{ Pa}$$

and the moisture content ω_2 is

$$\omega_2 = 0.622 \frac{p_{v2}}{p - p_{v2}} = 0.622 \times \frac{2982}{101325 - 2982} = 0.018861$$

The loss of water due to evaporation is

$$\dot{m}_{v2} - \dot{m}_{v1} = \dot{m}_{w3} - \dot{m}_{w4}$$

and since $\dot{m}_{a2} = \dot{m}_{a1}$,

$$\frac{\dot{m}_{w4}}{\dot{m}_{a1}} = \frac{\dot{m}_{w3}}{\dot{m}_{a1}} + \omega_1 - \omega_2$$

Energy equation

The energy equation for the tower is

$$(\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{w4}h_{w4}) - (\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{w3}h_{w3}) = 0$$

Since $\dot{m}_{a2} = \dot{m}_{a1}$, we can rewrite this equation as

$$\dot{m}_{a1}C_{p,a}(\theta_{a2} - \theta_{a1}) + (\dot{m}_{v2}h_{v2} - \dot{m}_{v1}h_{v1}) + (\dot{m}_{w4}h_{w4} - \dot{m}_{w3}h_{w3}) = 0$$

Dividing through by $\dot{m}_{a1} = \dot{m}_{a2}$ gives

$$C_{p,a}(\theta_{a2} - \theta_{a1}) + (\omega_2h_{v2} - \omega_1h_{v1}) + \frac{\dot{m}_{w4}}{\dot{m}_{a1}}h_{w4} - \frac{\dot{m}_{w3}}{\dot{m}_{a1}}h_{w3} = 0$$

Substituting the equation for $\dot{m}_{w4}/\dot{m}_{a1}$ gives

$$C_{p,a}(\theta_{a2} - \theta_{a1}) + (\omega_2h_{v2} - \omega_1h_{v1}) + \left(\frac{\dot{m}_{w3}}{\dot{m}_{a1}} + \omega_1 - \omega_2\right)h_{w4} - \frac{\dot{m}_{w3}}{\dot{m}_{a1}}h_{w3} = 0$$

or

$$C_{p,a}(\theta_{a2} - \theta_{a1}) + (\omega_2h_{v2} - \omega_1h_{v1}) + \frac{\dot{m}_{w3}}{\dot{m}_{a1}}(h_{w4} - h_{w3}) + (\omega_1 - \omega_2)h_{w4} = 0$$

From Eqn. (2.15), the specific enthalpy of the water vapour at station 1 h_{v1} is

$$h_{v1} = 2500.8 + 1.86\theta_1 = 2500.8 + 1.86 \times 15 = 2529 \text{ kJ kg}^{-1}$$

and the specific enthalpy of the water vapour at station 2 h_{v2} is

$$h_{v2} = 2500.8 + 1.86\theta_2 = 2500.8 + 1.86 \times 24 = 2545 \text{ kJ kg}^{-1}$$

The temperature of the warm water at station 3 is 26°C, so from Eqn. (2.2) the specific enthalpy of the water h_{w3} is

$$\begin{aligned} h_{w3} &= h_{f3} = a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5 \\ &= -2.844699 \times 10^{-2} + 4.211925 \times 26 - 1.017034 \times 10^{-3} \times 26^2 \\ &\quad + 1.311054 \times 10^{-5} \times 26^3 - 6.756469 \times 10^{-8} \times 26^4 + 1.724481 \times 10^{-10} \times 26^5 \\ &= 109.00 \text{ kJ kg}^{-1} \end{aligned}$$

The temperature of the cold water at station 4 is 12°C, so the specific enthalpy of the water h_{w4} is

$$\begin{aligned} h_{w4} &= h_{f4} = a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5 \\ &= -2.844699 \times 10^{-2} + 4.211925 \times 12 - 1.017034 \times 10^{-3} \times 12^2 \\ &\quad + 1.311054 \times 10^{-5} \times 12^3 - 6.756469 \times 10^{-8} \times 12^4 + 1.724481 \times 10^{-10} \times 12^5 \\ &= 50.39 \text{ kJ kg}^{-1} \end{aligned}$$

Thus $\dot{m}_{w3}/\dot{m}_{a1}$ is

$$\begin{aligned} \frac{\dot{m}_{w3}}{\dot{m}_{a1}} &= \frac{C_{p,a}(\theta_{a2} - \theta_{a1}) + (\omega_2 h_{v2} - \omega_1 h_{v1}) + (\omega_1 - \omega_2)h_{w4}}{(h_{w3} - h_{w4})} \\ &= \frac{1.005 \times (24 - 15) + (0.018861 \times 2545 - 0.005806 \times 2529) + (0.005806 - 0.018861) \times 50.39}{(109.00 - 50.39)} \\ &= \frac{9.045 + 33.318 - 0.658}{58.61} = 0.7116 \end{aligned}$$

Mass flow rate of moist air into the tower

Now that we have $\dot{m}_{w3}/\dot{m}_{a1}$, we can calculate the mass flow rate of dry air at station 1, \dot{m}_{a1} :

$$\dot{m}_{a1} = \frac{\dot{m}_{w3}}{0.7116} = \frac{1000}{0.7116} = 1405 \text{ kg min}^{-1}$$

The mass flow rate of water vapour at station 1, \dot{m}_{v1} , is

$$\dot{m}_{v1} = \omega_1 \dot{m}_{a1} = 0.005806 \times 1405 = 8.16 \text{ kg min}^{-1}$$

and so the mass flow rate of moist air at station 1 \dot{m}_1 is

$$\dot{m}_1 = \dot{m}_{a1} + \dot{m}_{v1} = 1405 + 8.16 = 1413 \text{ kg min}^{-1}$$

Rate of loss of cooling water

The mass flow rate of water vapour at station 2 \dot{m}_{v2} is

$$\dot{m}_{v2} = \omega_2 \dot{m}_{a2} = \omega_2 \dot{m}_{a1} = 0.018861 \times 1405 = 26.50 \text{ kg min}^{-1}$$

and so the rate of loss of cooling water is

$$\dot{m}_{v2} - \dot{m}_{v1} = 26.50 - 8.16 = 18.34 \text{ kg min}^{-1}$$

6 References

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